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Appendix D. Derivation of the invasion rates, Dij, given in Eq. 3.

To derive Eq. 3, we considered the two-species scenario where species *i* invades species *j*. We set $\beta_{ii} = 0$ and $\kappa_{ii} = 0$ to obtain a "worst-case" scenario, because positive values of β_{ii} and κ_{ii} facilitate coexistence, at least over the range of parameter values appropriate for our BCI data (Appendix A). In addition, we assumed that adult death rates are the same and set $d_i = d_j = d$. In the adult stage of the model (Eq. 2), invasion requires

$$\lim_{x_i \to 0} E \log\left[\frac{x_i(t+1)}{x_i(t)}\right] = \lim_{x_i \to 0} E \log\left[d + (1-d)\left[\frac{s_i(t)}{x_i(s_i(t)+s_j(t))}\right]\right] \qquad \text{(D.1)}$$

Assuming $d \to 1$, this becomes

$$\left(E\left[\frac{s_i(t)}{x_i(t)(s_i(t)+s_j(t))}\right]-1\right) > 0 \qquad \text{(D.2)}$$

and thus invasion requires $E\left[\frac{s_i(t)}{x_i(t)\left(s_i(t)+s_j(t)\right)}\right]>1$.

To specify $s_i(t)$ and $s_j(t)$, we first expanded equation 1 by summing over the lag τ time steps,

$$s_i(t)=\sum_{\tau=1}^{\infty} f_i^{\tau-1}\frac{R_i(t-\tau-1)x_i(t-\tau-1)}{1+\alpha_{ii}R_i(t-\tau-1)x_i(t-\tau-1)+\alpha_{ij}R_j(t-\tau-1)x_j(t-\tau-1)}. \qquad \text{(D.3)}$$

Substituting for $s_i(t)$ and $s_j(t)$, and recognizing that at the invasion boundary $\frac{x_i(t-\tau)}{x_i(t)}\approx 1$, leads to

$$E\left[\frac{s_i(t)}{x_i(t)\left(s_j(t)+s_i(t)\right)}\right]\approx E\left[\frac{\sum_{\tau=1}^{t} f_i^{\tau}\frac{R_i(t-\tau-1)}{1+\alpha_{ij}R_j(t-\tau-1)}}{\sum_{\tau=1}^{t} f_j^{\tau}\frac{R_j(t-\tau-1)}{1+\alpha_{jj}R_j(t-\tau-1)}}\right] \qquad \text{(D.4)}$$

This gives Eq. 3.